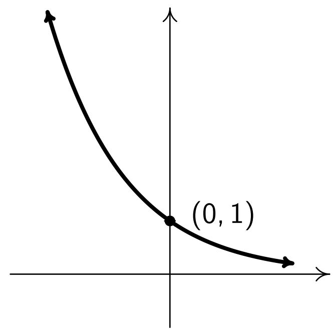
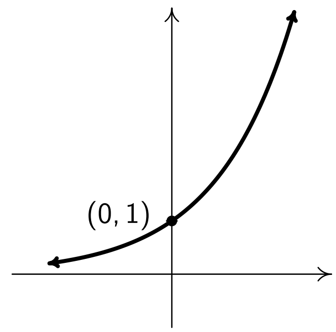
MATH 1650 SUMMARY OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

DEFINITION: An **exponential function with base** b is a function of the form $f(x) = b^x$ where $b > 0$, $b \neq 1$.

GRAPHS OF EXPONENTIAL FUNCTIONS: The graph of $f(x) = b^x$ resembles:



$0 < b < 1$ (Decay)



$b > 1$ (Growth)

- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- y-intercept: $(0, 1)$
- H.A.: $y = 0$

PROPERTIES OF EXPONENTIAL FUNCTIONS: Let $f(x) = b^x$:

- **ONE-TO-ONE:** $f(u) = f(w) \iff u = w$. i.e., $b^u = b^w \iff u = w$.

  'The outputs (exponential expressions) are the same if and only if the inputs (exponents) are the same.'

- **PRODUCT RULE:** $f(u)f(w) = f(u + w)$. i.e., $b^u b^w = b^{u+w}$

  'When multiplying with the same base, add the exponents of the factors.'

- **QUOTIENT RULE:** $\dfrac{f(u)}{f(w)} = f(u - w)$. i.e., $\dfrac{b^u}{b^w} = b^{u-w}$

  'When dividing with the same base, subtract: exponent on numerator − exponent on denominator.'

- **POWER RULE:** $f(u)^p = f(up)$. i.e., $(b^u)^p = b^{up}$

  'When taking a power to a power, multiply the exponents.'

DEFINITION: The **logarithm function base** b, denoted $\log_b(x)$ is the inverse of b^x. This means:

- The logarithm base b 'undoes' the exponential base b: $\log_b(b^x) = x$.
- The exponential base b 'undoes' the logarithm base b: $b^{\log_b(x)} = x$.

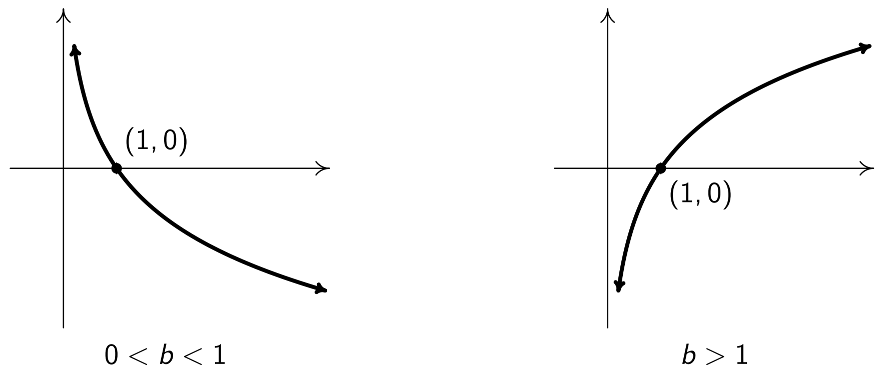
In other words: $\log_b(x)$ is the exponent you put on b to get x: $\log_2(8) = 3$ since $2^3 = 8$. In general:

$$\log_b(x) = p \iff b^p = x$$

NOTE: All the properties of logarithm functions are 'inherited' properties from exponential functions because:

LOGS ARE EXPONENTS!

GRAPHS OF LOGARITHMIC FUNCTIONS: The graph of $f(x) = \log_b(x)$ resembles:



- Domain: $(0, \infty)$
- Range: $(-\infty, \infty)$
- x-intercept: $(1, 0)$
- V.A.: $x = 0$

FAMOUS BASES:

- 'Common' Base: $\log_{10}(x) = \log(x)$
- 'Natural' Base: $\log_e(x) = \ln(x)$

PROPERTIES OF LOGARITHMIC FUNCTIONS: Let $f(x) = \log_b(x)$:

- **ONE-TO-ONE:** $f(U) = f(W) \iff U = W$. i.e., $\log_b(U) = \log_b(W) \iff U = W$.

  'The outputs (logs) are the same if and only if the inputs (arguments) are the same.' Logs are exponents!

  'The exponents are the same if and only if the exponential expressions are the same.'

- **PRODUCT RULE:** $f(UW) = f(U) + f(W)$. i.e., $\log_b(UW) = \log_b(U) + \log_b(W)$

  'The log of a product is the sum of the logs of the factors. ' Logs are exponents!

  'The exponent on a product is the sum of the exponents of the factors.'

- **QUOTIENT RULE:** $f\left(\frac{U}{W}\right) = f(U) - f(W)$. i.e., $\log_b\left(\frac{U}{W}\right) = \log_b(U) - \log_b(W)$

  'The log of a quotient is the difference: log of numerator − log of denominator. ' Logs are exponents!

  'The exponent on a quotient is the difference: exponent of numerator − exponent of denominator.'

- **POWER RULE:** $f\left(U^p\right) = pf(U)$. i.e., $\log_b\left(U^p\right) = p\log_b(U)$

  'The log of (expression to a power) is the power times the log.' Logs are exponents!

  'The exponent on an expression to a power is the power times the exponent of the expression.'

- **CHANGE OF BASE:**

  - Exponential Functions: $a^u = b^{u\log_b(a)}$
  - Logarithmic Functions: $\log_a(U) = \dfrac{\log_b(U)}{\log_b(a)}$